A NOTE ON ESTIMATION IN DOUBLE SAMPLING

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(Received : October, 1981)

Let $\bar{X}_t$ be the finite population means for variates x_t (t=0, 1, 2). Let $\bar{x}_2'$ be a first-phase sample (SRSWOR, of size n') mean on x_2 and $\bar{x}_t$ be a second-phase sub-sample (SRSWOR, of size $n<n'$) means on x_t (t=0, 1, 2). For $R=\bar{X}_0/\bar{X}_1$ and for $P=\bar{X}_0\bar{X}_1$, Singh [1] gave double-sample estimators $\hat{R}_1=\hat{R}\bar{x}_2/\bar{x}_2'$, $\hat{R}_2=\hat{R}\bar{x}_2'/\bar{x}_2$ and $\hat{P}_1=\hat{P}\bar{x}_2/\bar{x}_2'$, $\hat{P}_2=\hat{P}_2\bar{x}_2'/\bar{x}_2$ respectively (wheie $\hat{R}=\bar{x}_0/\bar{x}_1$, $\hat{P}=\bar{x}_0\bar{x}_1$). Their asymptotic mean square errors (MSE), to the first order of approximation, dervied using delta method, are vide Singh [1] :

$$MSE(\hat{R}_t)=M+fR^2c_2^2\,[1-2(-1)^t_c],\qquad t=1, 2$$

$$MS(E\hat{P}_t)=M_1+fP^2\,c_2^2\,[1-2(-1)^t c^*],\quad t=1, 2$$

where $\quad M=MSE(\hat{R});\ M_1=MSE(\hat{P}),\ f=\left(\frac{1}{n}-\frac{1}{n'}\right)$,

c_t=coefficient of variation of $x_t(t=0, 1, 2)$,

$\rho_{tt'}$=correlation coefficient between x_t and $x_{t'}$,

$(t,\ t'=0, 1, 2),\ c=\rho_{02}(c_0/c_2)-\rho_{12}(c_1/c_2)$

and $\quad c^*=\rho_{02}(c_0/c_2)+\rho_{12}(c_1/c)$.

Writing $s_{tt'}$ for the sum-sample (of size n)—covariance between x_t and $x_{t'}$ (for $t,\ t'=0$, 1, 2) we propose better alternative estimators respectively for R and P as

$$\hat{R}^*=\hat{R}+a(\bar{x}_2-\bar{x}_2'),$$

where $\quad a=-\frac{\hat{R}}{s_{22}}(s_{02}/\bar{x}_0-s_{12}/\bar{x}_1)$

and $$\hat{p}^* = \hat{p} + b(\bar{x}_2 - \bar{x}_2'),$$

where $$b = -\frac{\hat{p}}{s_{22}}(s_{02}/\bar{x}_0 + s_{12}/\bar{x}_1).$$

Their asymptotic (to first order of approximation) *MSE's* are using delta methods again, respectively,

$$M(\hat{R}^*) = M - fR^2 c_2^2 c_2 < MSE(\hat{R}_t)$$

$$\text{and } M(\hat{p}^*) = M_1 - fP^2\, c_2^2\, c_2 < MSE(\hat{p}_t) \text{ for both } t = 1, 2$$

AN ILLUSTRATION

For the example considered by Singh (1965), we have

$$MSE(\hat{R}) = k'\ (0.5318)$$

$$MSE(\hat{R}_1) = k'\ (0.\ 5.318) - k^*(1.42020)$$

$$MSE(\hat{R}_2) = k'\ (0.5318) - k^*(.2728)$$

$$MSE(\hat{R}^*) = k'\ (0.5318) - k^*(.3123)$$

where $$k' = \frac{(N-n)}{nN} kR_2,$$

$$k^* = \left(\frac{1}{n} - \frac{1}{n'}\right) kR^2,$$

$$k = \frac{N}{(N-1)}$$ and N is the number of blocks in the population.

From above, we see that $\hat{R}^*$ is the most efficient.

REFERENCE

[1] Singh, M.P. (1965) : On the estimation of ratio and product of the population parameters. *Sankhya,* **27** (B), 32-128.